A Simple and Usable Wake Vortex Encounter Severity Metric

Ivan De Visscher
Grégoire Winckelmans

WaPT-Wake Prediction Technologies
a spin-off company from Université catholique de Louvain (UCL)

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The RMC is the vortex induced rolling moment on the follower normalized by the follower parameters

$$RMC = \frac{M_v}{\frac{1}{2} \rho V_f^2 S_f b_f}$$

where $M_v$ is the induced rolling moment, $V_f$ is the flight speed, $S_f$ is the wing surface and $b_f$ is the wing span.
Wake Vortex Encounter Induced Rolling Moment

\[ M_v = \frac{\rho V_f^2}{2} C_{l,\alpha,eff} \int_{-b_f/2}^{b_f/2} c(y) \Delta \alpha(y) y \, dy \]

with \( C_{l,\alpha,eff} \) the “effective lift slope” coefficient, \( c(y) \) the chord distribution, and \( \Delta \alpha(y) = \frac{w_v(y)}{V_f} \) the relative increase of the angle of attack due to the vortex induced vertical velocity distribution along the span \( w_v(y) \):

\[ \Delta \alpha(y) = \frac{w_v(y)}{V_f} = \frac{\Gamma_{tot}}{V_f} \frac{1}{2\pi r} \frac{\Gamma(r)}{\Gamma_{tot}} \frac{(y - y_v)}{r} \]

With \( r = \sqrt{(y - y_v)^2 + z_v^2} \)
Rolling Moment Coefficient (RMC)

\[ RMC = \frac{\Gamma_{\text{tot}} \ C_{l,\alpha,\text{eff}}}{V_f b_f} \frac{1}{2\pi} \int_{-1}^{1} \frac{c(\eta) \ \Gamma(r)}{\bar{c} \ \Gamma_{\text{tot}}} \frac{(\eta - \eta_v)}{[(\eta - \eta_v)^2 + \zeta_v^2]} \eta \ d\eta \]

with \( \eta = \frac{y}{b_f}, \zeta = \frac{z}{b_f} \)

Hence:

\[ RMC = \frac{\Gamma_{\text{tot}} \ C_{l,\alpha,\text{eff}}}{V_f b_f} \frac{1}{2\pi} F \left( \frac{c(y)}{\bar{c}}, \frac{\Gamma(r)}{\Gamma_{\text{tot}}}, \frac{y_v}{b_f'}, \frac{z_v}{b_f} \right) \]
Rolling Moment Coefficient (RMC)

\[
RMC = \frac{\Gamma_{tot}}{V_f b_f} \frac{C_{l,\alpha,eff}}{2\pi} F \left( \frac{c(y)}{\bar{c}}', \frac{\Gamma(r)}{\Gamma_{tot}}', \frac{\gamma}{b'} \right)
\]

Assumption: centered wake vortex encounter
Wing chord distribution

Assumption: Elliptical chord distribution

\[
\frac{c(y)}{\bar{c}} = \frac{4}{\pi} \sqrt{1 - \left(\frac{y}{b_f/2}\right)^2}
\]

- Used for simplicity reason for RMC to be applicable to all aircraft types
- In RECAT-EU, the sensitivity of RMC to the detailed wing chord distribution was shown to be low
Rolling Moment Coefficient (RMC)

\[
RMC = \frac{\Gamma_{tot}}{V_f} \frac{C_{l,\alpha,eff}}{2\pi} \frac{\Gamma(r)}{\Gamma_{tot}} \frac{\gamma}{b} \frac{Z_\phi}{c}
\]

- Need to take into account the radial distribution of the circulation within the vortex
  - Indeed, typically 50-60% of the circulation lies within a radius of 5% of the generator wing span; the rest of the circulation lies beyond that radius, and with a long tail in the distribution (see, AGARDograph No. 204, “Vortex Wakes of Conventional Aircraft”, C. du P. Donaldson and A. Bilanin, 1975)
  - Hence, any distribution of circulation has a lengthscale that is proportional to the generator wing span
Circulation distribution models

- **Burnham-Hallock (B-H) model:**
  \[
  \frac{\Gamma(r)}{\Gamma_{tot}} = \frac{\left(\frac{r}{b_l}\right)^2}{\left[\left(\frac{r}{b_l}\right)^2 + a^2\right]}
  \]

- **Kaden-Winckelmans (K-W) model:**
  \[
  \frac{\Gamma(r)}{\Gamma_{tot}} = \frac{\alpha \left(\frac{r}{b_0/2}\right)^{1/2}}{1 + (\alpha-1)\left(\frac{r}{b_0/2}\right)}
  \]

- **Modified Betz model:**
  \[
  \frac{r}{b_l} = \frac{1}{8} \frac{(2\alpha \theta - \sin(2\alpha \theta))}{\sin(\alpha \theta)}, \quad \frac{\Gamma(r)}{\Gamma_{tot}} = \sin \theta
  \]
  with \(0 \leq \theta \leq \pi/2\)
B-H circulation distribution model

\[ \frac{\Gamma(r)}{\Gamma_{tot}} = \frac{\left(\frac{r}{b_l}\right)^2}{\left(\left(\frac{r}{b_l}\right)^2 + a^2\right)} \]

- Simple and commonly used in the WV literature
- The parameter \( a \) is here taken as 0.035 justified by:
  - Literature: typical values range from 0.03 to 0.05
  - Energy-based analysis
Determination of the \( a \) parameter by equating the kinetic energy of
- the near-wake cross-flow (determined from circulation distribution)
- to that of the resulting rolled-up two-vortex system made of B-H vortices

Example: Elliptic loading

\[
\begin{align*}
\frac{E_0}{\Gamma_0^2} &= \frac{\pi}{8} \\
\frac{s}{b_l} &= \frac{\pi}{4}
\end{align*}
\]

\( a = 0.0404 \)
Parameter of B-H vortex model

- **Clean configuration**: \( \frac{\Gamma(y)}{\Gamma_0} = \left(1 - \left(\frac{|y|}{b_l/2}\right)^p\right)^{1/p} \)

<table>
<thead>
<tr>
<th>Circulation distribution</th>
<th>( s )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptic (p=2)</td>
<td>( \pi/4 )</td>
<td>0.0404</td>
</tr>
<tr>
<td>Hyper-elliptic (p=2.5)</td>
<td>0.85</td>
<td>0.0276</td>
</tr>
<tr>
<td>Hyper-elliptic (p=3)</td>
<td>0.88</td>
<td>0.0189</td>
</tr>
</tbody>
</table>

- **Landing configuration**: \( \frac{\Gamma(y)}{\Gamma_0} = \begin{cases} 
\beta \left(1 - \left(\frac{|y|}{b_l/2}\right)^{p_1}\right)^{1/p_1}, & \text{if } |y| < \alpha \left(\frac{b_l}{2}\right) \\
\beta \left(1 - \left(\frac{|y|}{b_l/2}\right)^{p_1}\right)^{1/p_1} + (1 - \beta) \left(1 - \left(\frac{|y|}{\alpha \left(\frac{b_l}{2}\right)}\right)^{p_2}\right)^{1/p_2}, & \text{else}
\end{cases} \)

<table>
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<tr>
<th>Circulation distribution</th>
<th>( s )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double hyper-elliptic (p_1=2.5, p_2=3)</td>
<td>0.77</td>
<td>0.0366</td>
</tr>
<tr>
<td>Double hyper-elliptic (p_1=2.5, p_2=3.5)</td>
<td>0.78</td>
<td>0.0342</td>
</tr>
</tbody>
</table>

with \( \alpha = 0.75 \) and \( \beta = 0.6 \)
Correction Function to be used in the RMC

With the B-H vortex model the correction function can be obtained analytically (with $a = 0.035$):

$$F\left(\frac{b_l}{b_f}\right) = 1 - 2 \left(2a \frac{b_l}{b_f}\right)\left(\sqrt{1 + \left(2a \frac{b_l}{b_f}\right)^2} - \left(2a \frac{b_l}{b_f}\right)\right)$$
Sensitivity to the circulation distribution model

Case of an elliptical wing

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>$\frac{\Gamma(r = 0.05 b)}{\Gamma_{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-H</td>
<td>$\alpha = 0.0404$</td>
<td>0.605</td>
</tr>
<tr>
<td>K-W</td>
<td>$\alpha = 1.774$</td>
<td>0.576</td>
</tr>
<tr>
<td>Betz</td>
<td>$\alpha = 0.926$</td>
<td>0.560</td>
</tr>
</tbody>
</table>

Obtained correction function $F \left( \frac{b_l}{b_f} \right)$

<table>
<thead>
<tr>
<th>Model</th>
<th>$b/b_f=0.5$</th>
<th>$b/b_f=1.0$</th>
<th>$b/b_f=1.5$</th>
<th>$b/b_f=2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-H</td>
<td>0.922</td>
<td>0.851</td>
<td>0.785</td>
<td>0.725</td>
</tr>
<tr>
<td>K-W</td>
<td>0.913</td>
<td>0.829</td>
<td>0.754</td>
<td>0.694</td>
</tr>
<tr>
<td>Betz</td>
<td>0.917</td>
<td>0.837</td>
<td>0.762</td>
<td>0.696</td>
</tr>
</tbody>
</table>
Sensitivity to the circulation distribution model

Case of double hyper-elliptic with $p_1=2.5$, $p_2=3$

<table>
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<tr>
<th>Model</th>
<th>Parameter</th>
<th>$\frac{\Gamma(r = 0.05 b)}{\Gamma_{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-H</td>
<td>$\alpha = 0.0366$</td>
<td>0.650</td>
</tr>
<tr>
<td>K-W</td>
<td>$\alpha = 1.855$</td>
<td>0.601</td>
</tr>
<tr>
<td>Betz</td>
<td>$\alpha = 0.881$</td>
<td>0.585</td>
</tr>
</tbody>
</table>

Obtained correction function $F\left(\frac{b_l}{b_f}\right)$
Rolling Moment Coefficient (RMC)

\[
RMC = \frac{\Gamma_{tot}}{V_f b_f} \frac{C_{l,\alpha,eff}}{2\pi} F \left( \frac{b_l}{b_f} \right)
\]

Need to take into account the effective lift slope coefficient in case of wake vortex encounter
“Effective lift slope” in wake encounter: $C_{l,\alpha,\text{eff}}$

- Case of an elliptical wing in level flight

Uniform effective lift slope along the span (Prandtl correction)

$$C_{l,\alpha,\text{eff}} = C_{l,\alpha} \frac{AR}{AR + 2}$$

Note: the angles are not to scale as $\varepsilon_d \ll \alpha$.

$$\alpha_e = \alpha - \alpha_d \approx \frac{AR}{(AR + 2)} \alpha$$
Case of a wing in a wake encounter

« (...) for the rolling-moment coefficient, the functions (...) should be based on half of the span of the following wing. It is reasoned that the entire wing is in either an upwash or downwash for maximum lift, but when the maximum rolling moment occurs, only half of the wing is in an upwash and half is in a downwash. »

V. J. Rossow (1999), Lift-generated vortex wakes of subsonic transport aircraft, Progress in Aerospace Sciences 35, pp. 507–660

Applying Prandtl correction with half wing aspect ratio:

\[ AR_{\text{halfwing}} = \frac{1}{2} \frac{b_f}{\bar{c}} = \frac{1}{2} AR \]

\[ C_{l,\alpha,eff} = C_{l,\alpha} \frac{AR_{\text{halfwing}}}{AR_{\text{halfwing}} + 2} = C_{l,\alpha} \frac{1}{2} \frac{AR}{AR + 2} = C_{l,\alpha} \frac{AR}{AR + 4} \]
“Effective lift slope” in wake encounter: $C_{l,\alpha,eff}$

- Solving Prandtl integral equation
- Case of an elliptical wing with $AR$ from 5 to 20, encountering a B-H vortex with $a\frac{b_1}{b_f}$ from 2% to 8% and $C_{l,\alpha} = 2\pi$

Effective lift slope

A posteriori and global fit of the obtained induced RMC (with $C_{l,\alpha} = 2\pi$)

$$RMC = \frac{\Gamma_{tot}}{V_f b_f} \frac{AR}{(AR + C)} \frac{C_{l,\alpha}}{2\pi} F\left(\frac{b_l}{b_f}\right)$$

For the elliptical wing chord distribution, $C = 4.0$ was consistently obtained.

No sensitivity observed to the vortex parameters nor to the wing aspect ratio

This value is hence taken for the RMC metric
Sensitivity of the $C_{l,\alpha,eff}$ to the wing chord distribution

When varying the wing chord distribution and for $AR$ varying from 5 to 20, and for $\frac{a}{b_f} = 4\%$ one obtains $4.0 \leq C \leq 4.37$
Sensitivity of the $C_{l,\alpha,eff}$ to the vortex location

When moving the vortex to $\frac{y_v}{b_f} = 5\%$, for the various wing chord distributions, for $AR$ varying from 5 to 20, and for $a \frac{b_1}{b_f} = 4\%$, one obtains $3.99 \leq C \leq 4.38$
Assessment of the metric: $RMC = \frac{\Gamma_{tot}}{V_f b_f} \frac{AR}{(AR+4)} F \left( \frac{b_l}{b_f} \right)$

- **Reference data: Airbus flight encounter tests (2011)**
  - Leaders: A380 and A346 in landing configuration
  - Followers: A343 and A320
  - Database contains measurements of
    - Flight speed $V_f$
    - Air density $\rho$
    - Roll acceleration and corresponding rolling moment $M_v$
    - Estimated wake vortex circulation $\Gamma_{tot}$
    - Estimated minimum distance to vortices
  - Manual quality screening of the data by an independent expert panel (A380 ICAO pilot working group led by EASA)

- **Data selection for metric assessment**
  - High quality rated by expert panel
  - « Almost centered » encounters (distance to vortex core < 0.25 $b_f$)
  - Reasonable estimated circulation (compared to aircraft type)
Assessment of the metric: Rolling Moment calculated from RMC vs measurements

\[ M_v = RMC \times \left( \frac{1}{2} \rho V_f^2 S_f b_f \right) \]

with

\[ RMC = \frac{\Gamma_{tot}}{V_f b_f AR + 4} F \left( \frac{b_l}{b_f} \right) \]

Best linear fit coefficient: 0.98
Assessment of the metric: Rolling Moment calculated from leading order term of RMC vs measurements

Best linear fit coefficient: 1.71

\[ M_v = \frac{\Gamma_{tot}}{V_f b_f} \times \left( \frac{1}{2} \rho V_f^2 S_f b_f \right) \]

Correction function significantly improves the correlation with measured \( M_v \)
Assessment of the metric: Rolling Moment Coefficient from metric vs from measurements

Mean deviation: -0.0009
RMS deviation: 0.0226

\[
RMC = \frac{\Gamma_{tot}}{V_f b_f} \frac{AR}{AR + 4} F \left( \frac{b_l}{b_f} \right)
\]
Assessment of the metric: Rolling Moment Coefficient from leading order term of metric vs from measurements

Mean deviation: 0.0635
RMS deviation: 0.0777

\[ RMC = \frac{\Gamma_{tot}}{V_f b_f} \]

When using only the leading order term, differences between aircraft pairs, that are not observed in the experiments, are created.
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