ON THE LOSS OF ROLL CONTROL
INDUCED BY THE WAKE VORTEX HAZARD

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Rolling Moment Induced on the Following Aircraft

\[ dL(y) = \frac{1}{2} C_{\text{l}} \rho U w(y) c(y) \ dy \]

\[ R = \int_{-b/2}^{b/2} y \ dL(y) \]

\[ R = -\frac{1}{2} C_{\text{l}} \rho U \int_{-b/2}^{b/2} y \ c(y) w(y) \ dy \quad (2) \]
Wing Geometry: Sweptback (of following aircraft)

root chord: \(c_r\)

tip chord: \(c_i\)

average chord: \(2\bar{c} = c_i + c_r\)

taper ratio: \(\lambda \equiv \frac{c_i}{c_r}\)

Chord as a function of span

\[
c(y) = c_r + (c_i - c_r) \frac{2|y|}{b}
\]

alternatively:

\[
c(y) = \frac{2\bar{c}}{1 + \lambda} \left(1 + \frac{\lambda - 1}{b} 2|y|\right)
\] (2)
Counter-rotating wing tip vortices (from leading aircraft) 1/2

One Hallock-Burnham (HB) vortex

\[ w(r) = \frac{\Gamma_0}{2\pi} \frac{r}{r^2 + a^2} \]

- Tangential velocity
- Reference calculation
- Radius
- Vortex core radius
Counter-rotating wing tip vortices (from leading aircraft) 2-2

**Peak**

\[ w_0(r) = w(a) = \frac{\Gamma_0}{4\pi a} = \frac{\Omega a}{4} \]

\[ w_0(r) = \frac{\Omega}{2} \frac{a^2 r}{r^2 + a^2} \]

**Small r:** \[ w_0(r) \sim \frac{\Omega r}{2} \] Rigid body

**Large r:** \[ w_0(r) \sim \frac{\Gamma_0}{4\pi r} \] Potential vortex

**Two counter-rotating HB Vortices**

\[ 2w(y) = \Omega_r a_r^2 \frac{y - y_r}{a_r^2 + (y - y_r)^2} - \Omega_l a_l^2 \frac{y - y_l}{a_l^2 + (y - y_l)^2} \]

(3)

**Vorticity:** \( \Omega_r, -\Omega_l \)

**Radius:** ar, al

**Position:** \( y_r, y_l \)
Substitution (2) and (3) in (1) rolling moment

\[ 2R = -\frac{C_{Lr} \rho U \Omega_r a_r^2}{1 + \lambda} \int_{-b/2}^{b/2} y \left( 1 + \frac{\lambda - 1}{b} y \right) \frac{y - y_r}{a_r^2 + (y - y_r)^2} dy + \text{same for } (-\Omega_r, a_r, y_r) \]

After evaluation of integrals

\[ R = -\frac{2}{1 + \lambda} C_{Lr} \rho U_2 S a^2 \Omega(t) \]

Encounter Parameters: \( h_r, h_l, h \)

\[ \Omega_r a_r^2 h_r - \Omega_l a_l^2 h_l = (\Omega_r - \Omega_l) a^2 h \]
Vorticity as a Function of Distance (from leading aircraft)

1 – Leading aircraft trails two wing tip vortices.

2 – Wing tip vortices curl-up: vorticity increases.

3 – Weak atmospheric dissipation limits vorticity growth: reaches peak.

4 – Vorticity decays at larger distance behind leading aircraft ≡ longer time to reach following aircraft
Vorticity as a Function of Time (at following aircraft)

– Wake Vortex Strength of Leading Aircraft

\[ \Gamma_0 = \frac{c_W}{\rho U_1 S_1} \]

Leading Aircraft that creates the Wake

– Vorticity at Following Aircraft

\[ \Omega(t) = \frac{\Gamma_0}{2\pi\eta t} \exp \left( -\frac{a^2}{2\eta t} \right) \]

Atmospheric diffusivity – vortex decay

– Peak Vorticity

\[ \Omega_{\text{max}} = \Omega_{\text{max}}(t_{\text{max}}) = \frac{\Gamma_0}{\pi a^2 e} \]

occurs at time

\[ t_* = \frac{a^2}{2\eta} \]

larger vortex radius

smaller dissipation

WORST ENCOUNTER CONDITION
Roll Motion of the Following Aircraft

\[ I_2 \ddot{\phi} = m_2 \left( r_2 \right)^2 \ddot{\phi} = -\frac{1}{2} \rho U_2 S_2 \left( b_2 \right)^2 C_{\delta}\phi \]

\[ + \rho S_2 b_2 \left( U_2 \right)^2 C_{\delta} \delta(t) \]

\[ - \frac{2h}{1 + \lambda} \frac{C_{\text{e}_2}}{2\pi} \frac{U_2 S_2}{U_1 S_1} C_{\text{r}_2} \frac{a^2}{\eta t} \exp \left( -\frac{a^2}{2\eta t} \right) \]

Rolling moment on following aircraft induced by the wake of the leading aircraft

\text{INERTIA} = - \text{DAMPING} + \text{CONTROLS} + \text{WAKE VORTEX}
CASE I – Sufficient Control Power to Counter Wake Effects (safe case)

INDUCED ROLLING MOMENT = AILERON CONTROLS MOMENT

AILERON DEFLECTION (OF FOLLOWING AIRCRAFT)
THAT COMPENSATES WAKE VORTEX (OF LEADING AIRCRAFT)

\[
\delta(t) = \frac{1}{C_\delta} \frac{2h}{1 + \frac{\lambda}{2\pi}} \frac{C_{L_{\alpha}}}{\rho S_1 U_1 U_2} \frac{W}{b_2} \frac{c_{r_1} a^2}{\eta t} \exp \left( -\frac{a^2}{2\eta t} \right)
\]

INCREASES WITH:
- smaller aileron \( C_\delta \)
- \textit{larger mean encounter parameter} \( h \)
- smaller taper ratio \( \lambda \)
- \textit{larger lift slope} \( C_{L_{\alpha}} \)
- smaller air density \( \rho \) (weaker aerodynamic forces)
- \textit{larger wing loading of leading aircraft:} \( W_1 / S_1 \)
- smaller velocity of both aircraft \( U_1, U_2 \) (need larger wake vortex strength)
- \textit{larger root chord of leading aircraft} \( c_{r_1} \) (larger wake vortex strength)
- smaller span of following aircraft \( b_2 \) : less aileron moment arm;
- \textit{larger vortex core radius squared:} \( a^2 \)
- smaller dissipation \( \eta \) : less vortex decay.
CASE II – Free Aileron Response

\[ \dot{\phi}_f + \mu \dot{\phi}_f = 0 \]

\( \phi_f(t) = \phi_0 + \frac{\dot{\phi}_0}{\mu} \left(1 - e^{-\mu t}\right) \)

**DIMENSIONLESS DAMPING COEFFICIENT**

\[ \bar{\mu} = \frac{1}{2} \frac{\rho U_2 S_2}{W_2/g} \left(\frac{b_2}{r_2}\right)^2 C_\phi \]

**INCREASES WITH:**
- larger air density \( \rho \): lower altitude and larger aerodynamic forces;
- smaller wing loading \( W_2 / S_2 \): light aircraft more easily damped;
- larger airspeed \( U_2 \): larger forces.
- larger ratio of span \( b_2 \) to gyration radius \( r_2 \) squared: \( \left(\frac{b_2}{r_2}\right)^2 \)
- larger damping coefficient \( C_\phi \)
CASE III – Forced Response to Aileron Deflection

\[ \ddot{\phi}_e + \bar{\mu} \dot{\phi}_e = \nu \]

NO WAKE VORTEX

AILERON FORCING:

\[ \nu = \frac{\rho S_2 b_2}{g W_2} \left( \frac{U_2}{r_2} \right)^2 C_\delta \delta_{\max} \]

INCREASES WITH:
- larger air density: \( \rho \)
- lower wing loading \( \frac{S_2}{W_2} \)
- larger span \( b \), more effective ailerons;
- larger ratio of airspeed to gyration radius squared: larger ratio of aerodynamic forces to inertia forces;
- larger aileron coefficient \( C_\delta \)
- larger aileron deflection \( \delta_{\max} \)
Total Bank Angle Response:

\[ \phi(t) = \phi_f(t) + \phi_c(t) + \phi_w(t) \]

CONTROL RESPONSE

\[ \phi_c(t) = 2 \frac{U_2}{b_2} \frac{C_\delta}{C_\phi} \delta_{\text{max}} t \]

BANK ANGLE INCREASES WITH:

- larger airspeed \( U_2 \) : larger aerodynamic forces.

- smaller span: less roll inertia;

- larger aileron coefficient: \( C_\delta \) more roll control power;

- smaller roll damping coefficient: \( C_\phi \) less damping to oppose roll;

- larger aileron deflection \( \delta_{\text{max}} \)
Case IV: Wake Vortex Response

\[ \ddot{\phi}_w + \mu \dot{\phi}_w = -\frac{\xi}{t} \exp \left( -\frac{a^2}{2\eta t} \right) \]

**VORTEX EFFECT**

\[ \xi = \frac{2h \ C_{L_\infty} \ W_1 / S_1 \ U_2 \left( \frac{a}{r_2} \right)^2 \ c_n \ g}{1 + \lambda \frac{2\pi \ W_2 / S_2 \ U_1}{\eta}} \]

**INCREASES WITH:**
- larger encounter parameter \( h \);
- smaller taper ratio: rectangular more susceptible to vortex than swept/delta wing
- larger lift slope: more lift and roll
- larger ratio of wing loading of a/c 1 to wing loading to a/c 2: light a/c behind heavy a/c is worst case
- larger ratio of airspeed of a/c 2 to a/c 1 because a/c 2 catches wake of a/c 1 sooner;
- larger ratio of vortex core radius \( a \) to gyration radius \( r_2 \) squared;
- larger root chord of a/c 1
- less vortex decay: \( \eta \)
Case IV: Wake Vortex Response in terms of Exponential Integrals

DIMENSION LESS TIME: TIME $t$ DIVIDED BY TIME $t_{\text{max}}$ OF PEAK VORTICITY

$$\tau \equiv \frac{t}{t_{\text{max}}} = \frac{2\eta t}{a^2}$$

INCREASES FOR:
- stronger vortex decay: $\eta$
- smaller vortex core radius: $a$

ROLL RATE: $\Phi(\tau) \equiv \phi_w(t)$

$$\Phi(\tau) = -\xi e^{-\mu\tau} \left[ E_0 \left( \frac{1}{\tau} \right) + \sum_{n=1}^{\infty} \frac{\mu^n}{n!} E_n \left( \frac{1}{\tau} \right) \right]$$

WHERE:
- vortex factor: $\xi \equiv \frac{a^2}{2\eta}$
- damping: $\mu \equiv \bar{\mu} \frac{a^2}{2\eta}$

- generalized exponential integral: $E_n \left( \frac{1}{\tau} \right) \equiv \int_0^{1/\tau} e^{-1/\tau} \tau^{n-1} d\tau$
Roll Rate and Bank Angle

Dimensionless form valid for all aircraft pairs

- Peak roll rate
- Roll up
- Decay
- Asymptotic bank angle
Special (B757-200) Aircraft Leading and Following: s-s

Peak roll rate: 2.35°/s

Asymptotic bank angle 12.4°
Without and with Roll Damping

\[ \phi \text{[°]} \]

\[ t \text{[s]} \]

\( \mu = 0 \)  No damping

\( \mu \neq 0 \)  damping
Weak and Strong Roll Damping

Weak and strong roll damping

\[ \phi^\circ \]

\[ t[s] \]

0.25 = \( \mu \)

0.375

0.50

0.75

1.0

MORE DAMPING SMALLER BANK ANGLE
Special (s) following aircraft

Leading Aircraft

l – light: Jetstar
m – medium: B737 – 200
s – special; B757 – 200
h – heavy: B747 – 100
v – very large: A380 – 100
Heavy (h) following aircraft

Leading Aircraft

l – light: Jetstar
m – medium: B737 – 200
s – special; B757 – 200
h – heavy: B747 – 100
v – very large: A380 – 100
Light (l) following aircraft

**Leading Aircraft**

- **l** – light: Jetstar
- **m** – medium: B737 – 200
- **s** – special; B757 – 200
- **h** – heavy: B747 – 100
- **v** – very large: A380 – 100
### Table I – Wake vortex Effects on Following Special Aircraft

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Lead</th>
<th>Roll rate</th>
<th>Bank angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Peak value (°/s)</td>
<td>Peak time (s)</td>
</tr>
<tr>
<td>P e e k</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>light (l)</td>
<td></td>
<td>1.17488</td>
<td>1.14958</td>
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<tr>
<td>medium (m)</td>
<td></td>
<td>1.64483</td>
<td>2.94292</td>
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<tr>
<td>special (s)</td>
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<td>2.34975</td>
<td>4.59831</td>
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<tr>
<td>heavy (h)</td>
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<td>3.05468</td>
<td>7.77115</td>
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<tr>
<td>very-large (v)</td>
<td></td>
<td>4.69951</td>
<td>18.3932</td>
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</tbody>
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Table II – Wake vortex Effects on Following Heavy Aircraft

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Roll rate</th>
<th>Bank angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak value (°/s)</td>
<td>Peak time (s)</td>
</tr>
<tr>
<td>Following</td>
<td>Leading</td>
<td>Light (l)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Medium (m)</td>
</tr>
<tr>
<td>Heavy (h)</td>
<td></td>
<td>Special (s)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Heavy (h)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Very-large (v)</td>
</tr>
</tbody>
</table>
Table III – Wake vortex Effects on Following Light Aircraft

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Roll rate</th>
<th>Bank angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak value (°/s)</td>
<td>Peak time (s)</td>
</tr>
<tr>
<td>Following</td>
<td>Leading</td>
<td></td>
</tr>
<tr>
<td>light (l)</td>
<td>light (l)</td>
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<td>medium (m)</td>
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<td></td>
<td>special (s)</td>
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<tr>
<td></td>
<td>heavy (h)</td>
<td>12.1914</td>
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<tr>
<td></td>
<td>very-large (v)</td>
<td>18.756</td>
</tr>
</tbody>
</table>
CONCLUSIONS (1/2)

• The roll response due to the wake vortex effects can be calculated for any pair of aircraft;

• The response is the same for all pairs of aircraft using suitable dimensionless variables;

• Using the parameters for each aircraft specifies the dimensionless roll rate and bank angle as a function of time;

• Peak roll rate and the time of its occurrence can be calculated;
CONCLUSIONS (2/2)

• Maximum bank angle can be determined and depends on damping to stop the rolling motion;

• Initial conditions (initial bank angle and roll rate) can be combined with aileron deflection and wake vortex effects;

• Wake vortex effects can be calculated for any separation between leading and following aircraft;

• Parametric dependences on aircraft characteristics and flight conditions are identified;

• Atmospheric model is simplified and is main limitation.