ON THE LOSS OF ROLL CONTROL INDUCED BY THE WAKE VORTEX HAZARD

by

L.M.B.C. Campos & J.M.G. Marques

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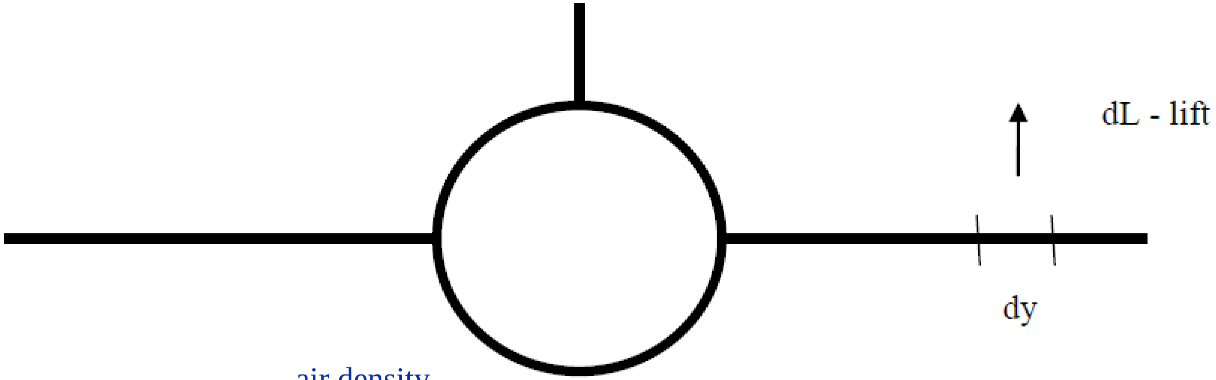
Rolling Moment Induced on the Following Aircraft



air density

downwash

per unit span

$$dL(y) = \frac{1}{2} C_{L_\infty} \rho U\, w(y)\, c(y)\, dy$$

lift

lift slope

airspeed velocity

wing chord

lift per unit span

$$R = -\int_{-b/2}^{b/2} y\ \mathrm{d}L(y)$$

Rolling moment
wake vortex effect

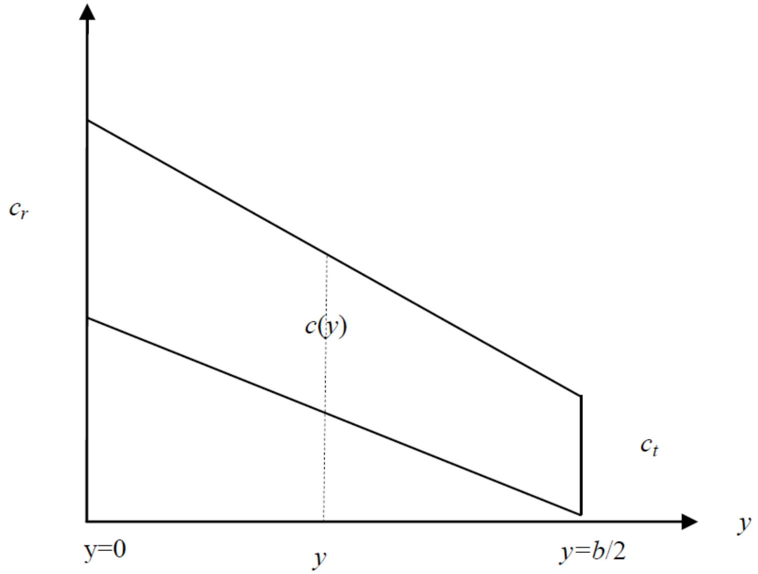
moment arm

$$R = -\frac{1}{2} C_{L_\alpha} \rho U \int_{-b/2}^{b/2} y\ c(y)\, w(y)\ dy \qquad \textbf{(2)}$$

Wing planform of following aircraft

Downwash of leading aircraft

Wing Geometry: Sweptback (of following aircraft)



root chord: c_r

tip chord: c_t

average chord: $2\overline{c} = c_t + c_r$

taper ratio: $\lambda \equiv \frac{c_t}{c_r}$

Chord as a funtion of span

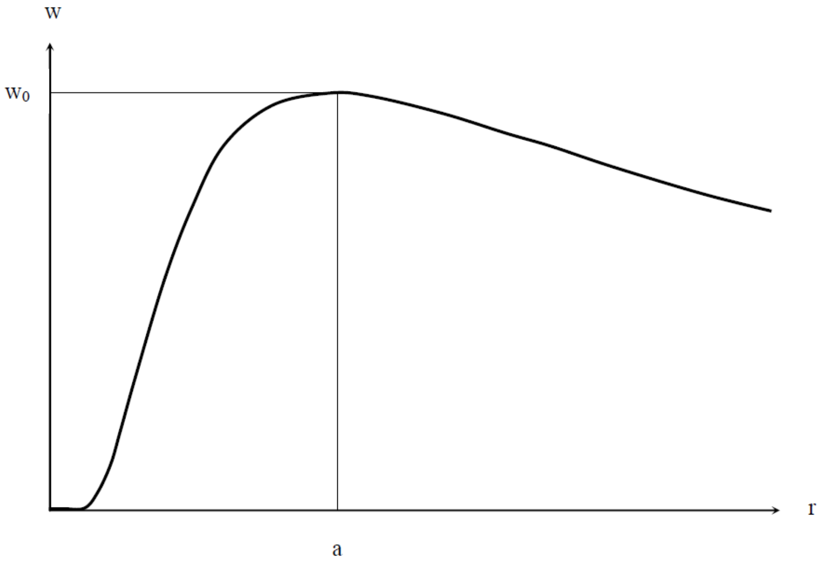
$$c(y) = c_r + (c_t - c_r)\frac{2|y|}{b}$$

alternatively:

$$c(y) = \frac{2\overline{c}}{1+\lambda}\left(1 + \frac{\lambda - 1}{b}2|y|\right) \qquad \textbf{(2)}$$

Counter-rotating wing tip vortices (from leading aircraft) 1/2

One Hallock-Burnham (HB) vortex



tangential velocity

Reference calculation

$$w(r) = \frac{\Gamma_0}{2\pi} \frac{r}{r^2 + a^2}$$

radius

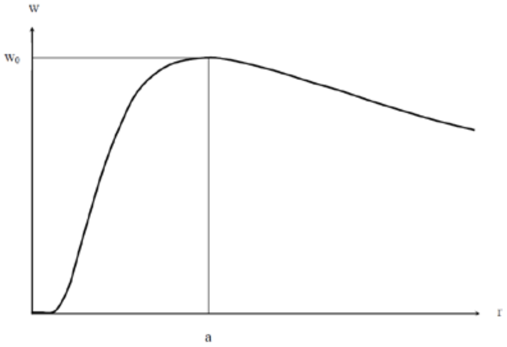
Vortex core radius

Counter-rotating wing tip vortices (from leading aircraft) 2-2

Peak $$w_0(r) = w(a) = \frac{\Gamma_0}{4\pi a} = \frac{\Omega a}{4}$$

Reference vorticity

$$w_0(r) = \frac{\Omega}{2}\frac{a^2 r}{r^2 + a^2}$$



Small r: $$w_0(r) \sim \frac{\Omega r}{2}$$ **Rigid body**

Large r: $$w_0(r) \sim \frac{\Gamma_0}{4\pi r}$$ **Potential vortex**

Two counter-rotating HB Vortices

\+ RIGHT - LEFT

$$2w(y) = \Omega_r a_r^2 \frac{y - y_r}{a_r^2 + (y - y_r)^2} - \Omega_\ell a_\ell^2 \frac{y - y_\ell}{a_\ell^2 + (y - y_\ell)^2} \qquad \textbf{(3)}$$

Vorticity: $\Omega_r, -\Omega_\ell$ **Radius:** ar, al **Position:** y_r, y_ℓ

Rolling Moment Dependence on Encounter Conditions

Substitution (2) and (3) in (1) rolling moment

$$2R = -\frac{C_{L_\infty}\rho U\bar{c}\Omega_r a_r^2}{1+\lambda}\int_{-b/2}^{b/2} y\left(1+\frac{\lambda\text{-}1}{\text{b}}2\left|y\right|\right)\frac{y-y_r}{a_r^2+\left(y-y_r\right)^2}dy+\text{same for}\left(-\Omega_\ell, a_\ell, y_\ell\right)$$

After evaluation of integrals

lift slope

mass density

vortex core radius

$$R = -\frac{2}{1+\lambda}C_{L_\infty}\rho U_2 S_2 a^2 h\Omega\left(t\right)$$

Vorticity function

- varies with time (or distance from leading aircraft)

Taper ratio

airspeed

wing area

of following aircraft (index "2")

Encounter Parameters: h_r, h_ℓ, h

$$\Omega_r a_r^2 h_r - \Omega_\ell a_\ell^2 h_\ell = \left(\Omega_r - \Omega_\ell\right)a^2 h$$

right **left** **total**

vortex

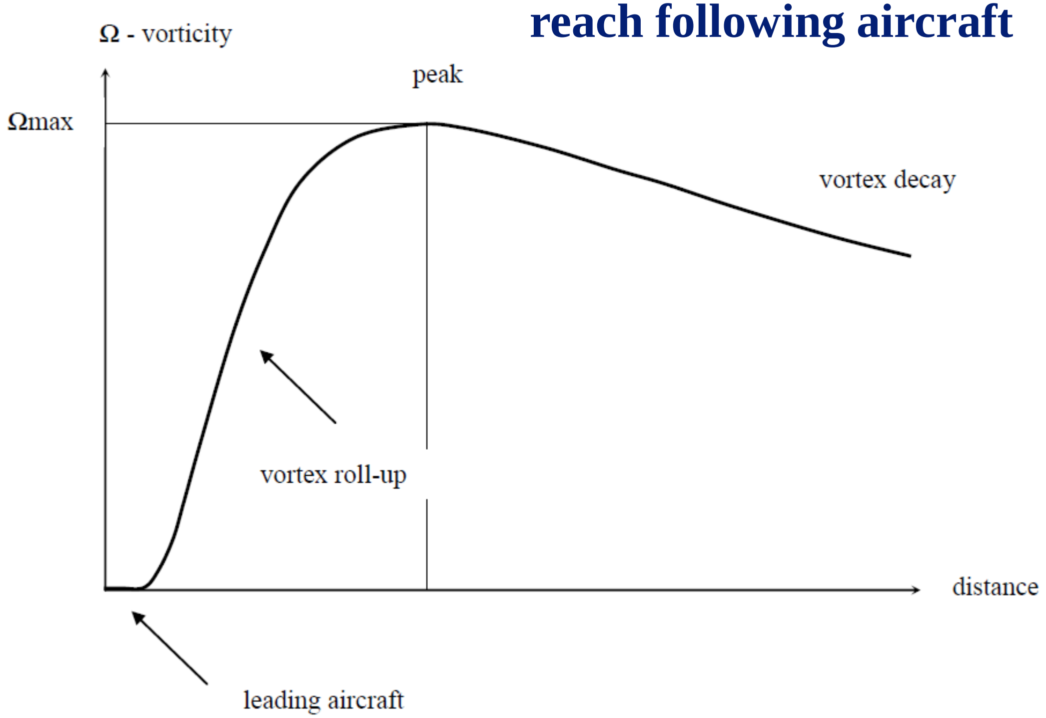
Vorticity as a Function of Distance (from leading aircraft)

1 – Leading aircraft trails two wing tip vortices.

3 – Weak atmospheric dissipation limits vorticity growth: reaches peak.

2 – Wing tip vortices curl-up: vorticity increases.

4 – Vorticity decays at larger distance behind leading aircraft ≡ longer time to reach following aircraft



Vorticity as a Function of Time (at following aircraft)

– Wake Vortex Strenght of Leading Aircraft

root chord

weight

$$\Gamma_0 = \frac{c_{r_1} W_1}{\rho U_1 S_1}$$

mass density

airspeed

wing area

Leading Aircraft that creates the Wake

– Vorticity at Following Aircraft

wake vortex strength of leading aircraft

$$\Omega(t) = \frac{\Gamma_0}{2\pi\eta t}\exp\left(-\frac{a^2}{2\eta t}\right)$$

Atmospheric diffusivity – vortex decay

a - vortex core radius

t - time

– Peak Vorticity

$$\Omega_{max} = \Omega_{max}\left(t_{max}\right) = \frac{\Gamma_0}{\pi a^2 e}$$

→

occurs at time

larger vortex radius

$$t_* = \frac{a^2}{2\eta}$$

smaller dissipation

WORST ENCOUNTER CONDITION

Roll Motion of the Following Aircraft

Moment of inertia

mass

Gyration radius

Roll acceleration

$$I_2\ddot{\phi} = m_2\left(r_2\right)^2\ddot{\phi}$$

INERTIA TERM

Roll rate

$$= -\frac{1}{2}\rho U_2 S_2\left(b_2\right)^2 C_{\dot{\phi}}\dot{\phi}$$

DAMPING TERM

Roll damping coefficient

$$+\rho S_2 b_2\left(U_2\right)^2 C_{\delta}\,\delta\left(t\right)$$

ROLL CONTROL TERM

$$-\frac{2h}{1+\lambda}\frac{C_{L_\infty}}{2\pi}W_1\frac{U_2}{U_1}\frac{S_2}{S_1}c_{r_1}\frac{a^2}{\eta t}\exp\left(-\frac{a^2}{2\eta t}\right)$$

WAKE EFFECT

Rolling moment on following aircraft induced by the wake of the leading aircraft

INERTIA = - DAMPING + CONTROLS + WAKE VORTEX

CASE I – Sufficient Control Power to Counter Wake Effects (safe case)

INDUCED ROLLING MOMENT = AILERON CONTROLS MOMENT

AILERON DEFLECTION (OF FOLLOWING AIRCRAFT) THAT COMPENSATES WAKE VORTEX (OF LEADING AIRCRAFT)

$$\delta(t)=\frac{1}{C_\delta}\frac{2h}{1+\lambda}\frac{C_{L_\infty}}{2\pi}\frac{W}{\rho S_1 U_1 U_2}\frac{c_{r_1}}{b_2}\frac{a^2}{\eta t}\exp\left(-\frac{a^2}{2\eta t}\right)$$

INCREASES WITH:

- smaller aileron C_δ
- *larger mean encounter parameter* h
- smaller taper ratio λ
- *larger lift slope* C_{L_∞}
- smaller air density ρ (weaker aerodynamic forces)
- *larger wing loading of leading aircraft:* W_1 / S_1
- smaller velocity of both aircraft U_1, U_2 (need larger wake vortex strength)
- *larger root chord of leading aircraft* c_{r_1} *(larger wake vortex strength)*
- smaller span of following aircraft b_2 : less aileron moment arm;
- *larger vortex core radius squared:* a^2
- smaller dissipation η : less vortex decay.

CASE II – Free Aileron Response

$$\ddot{\phi}_f + \bar{\mu}\,\dot{\phi}_f = 0$$

inertia

damping

NO CONTROLS
NO WAKE

time

Initial roll rate

$$\phi_f\left(t\right) = \phi_0 + \frac{\dot{\phi}_0}{\bar{\mu}}\left(1 - e^{-\bar{\mu}t}\right)$$

Bank angle at time *t*

Initial bank rate

DIMENSIONLESS DAMPING COEFFICIENT

$$\bar{\mu} \equiv \frac{1}{2}\frac{\rho U_2 S_2}{W_2 / g}\left(\frac{b_2}{r_2}\right)^2 C_{\dot{\varphi}}$$

INCREASES WITH:

- *larger air density* ρ*: lower altitude and larger aerodynamic forces;*
- *smaller wing loading* W_2 / S_2*: light aircraft more easily damped;*
- *larger airspeed* U_2*: larger forces.*
- *larger ratio of span* b_2 *to gyration radius* r_2 *squared:* $\left(b_2 / r_2\right)^2$
- *larger damping coefficient* $C_{\dot{\phi}}$

CASE III – Forced Response to Aileron Deflection

$$\ddot{\phi}_c + \bar{\mu}\,\dot{\phi}_c = \nu$$

NO WAKE VORTEX

AILERON FORCING:

$$\nu = \frac{\rho}{g}\frac{S_2}{W_2} b_2 \left(\frac{U_2}{r_2}\right)^2 C_\delta \delta_{\max}$$

INCREASES WITH:

- larger air density: ρ

- lower wing loading S_2 / W_2

- larger span *b*, more effective ailerons;

- larger ratio of airspeed to gyration radius squared: larger ratio of aerodynamic forces to inertia forces;

- larger aileron coefficient C_δ

- larger aileron deflection $\delta_{\max}$

Sum of Three Roll Responses

Total Bank Angle Response:

$$\phi(t)=\underset{\substack{\nwarrow\\ \text{free}}}{\phi_f}(t)+\underset{\substack{\nwarrow\\ \text{controls}}}{\phi_c}(t)+\phi_w(t)\underset{\substack{\nwarrow\\ \text{wake}}}{}$$

CONTROL RESPONSE

$$\phi_c(t)=2\frac{U_2}{b_2}\frac{C_\delta}{C_{\dot{\phi}}}\delta_{\max}t$$

BANK ANGLE INCREASES WITH:

- *larger airspeed* U_2 *: larger aerodynamic forces.*

- *smaller span: less roll inertia;*

- *larger aileron coefficient:* C_δ *more roll control power;*

- *smaller roll damping coefficient:* $C_{\dot{\phi}}$ *less damping to oppose roll;*

- *larger aileron deflection* $\delta_{\max}$

Case IV: Wake Vortex Response

$$\ddot{\phi}_w + \bar{\mu}\dot{\phi}_w = -\frac{\bar{\xi}}{t}\exp\left(-\frac{a^2}{2\eta t}\right)$$

VORTEX EFFECT

$$\bar{\xi} \equiv \frac{2h}{1+\lambda}\frac{C_{L_\infty}}{2\pi}\frac{W_1/S_1}{W_2/S_2}\frac{U_2}{U_1}\left(\frac{a}{r_2}\right)^2\frac{c_{r_1}g}{\eta}$$

INCREASES WITH:

- larger encounter parameter h;

- smaller taper ratio: rectangular more susceptible to vortex than swept/delta wing

- larger lift slope: more lift and roll

- larger ratio of wing loading of a/c 1 to wing loading to a/c 2: light a/c behind heavy a/c is worst case

- larger ratio of airspeed of a/c 2 to a/c 1 because a/c 2 catches wake of a/c 1 sooner;

- larger ratio of vortex core radius a to gyration radius r_2 squared;

- larger root chord of a/c 1

- less vortex decay: η

Case IV: Wake Vortex Response in terms of Exponential Integrals

DIMENSION LESS TIME: TIME t DIVIDED BY TIME tmax OF PEAK VORTICITY

$$\tau \equiv \frac{t}{t_{\max}} = \frac{2\eta t}{a^2}$$

INCREASES FOR:

- stronger vortex decay: η
- smaller vortex core radius: a

ROLL RATE: $\Phi(\tau) \equiv \phi_w(t)$

$$\dot{\Phi}(\tau) = -\xi e^{-\mu\tau} \left[E_0(1/\tau) + \sum_{n=1}^{\infty} \frac{\mu^n}{n!} E_n(1/\tau) \right]$$

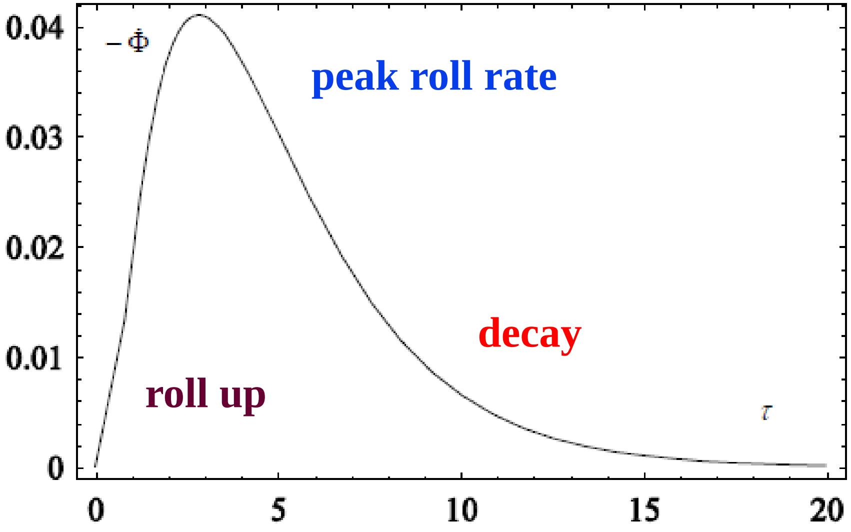
WHERE:

- vortex factor: $\xi \equiv \bar{\xi} \frac{a^2}{2\eta}$ - damping: $\mu \equiv \bar{\mu} \frac{a^2}{2\eta}$

- generalized exponential integral: $E_n\left(\frac{1}{\tau}\right) \equiv \int_0^{1/\tau} e^{-1/\tau} \tau^{n-1} d\tau$

Roll Rate and Bank Angle

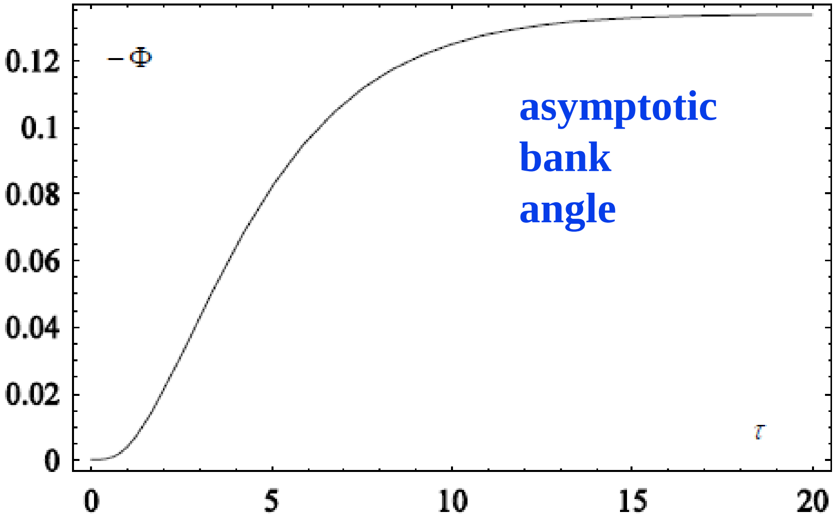
Dimensionless form valid for all aircraft pairs



peak roll rate

decay

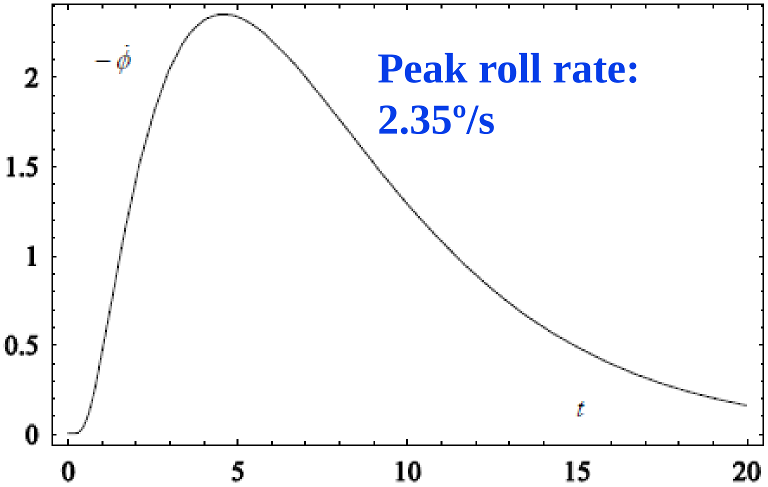
roll up



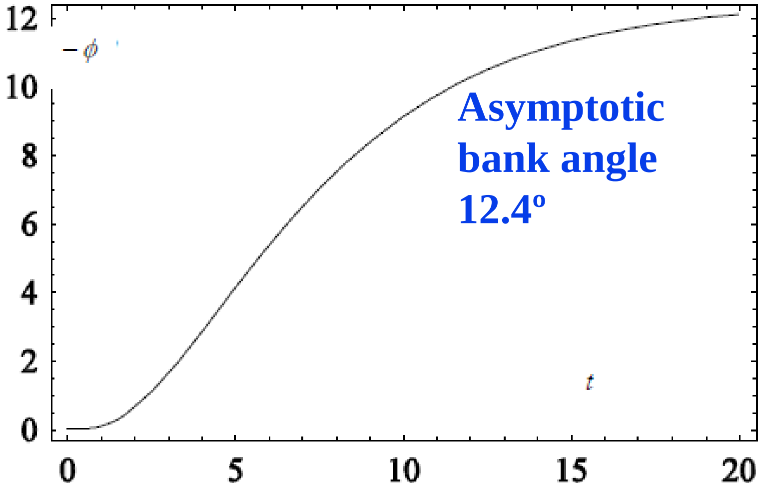
asymptotic bank angle

Special (B757-200) Aircraft Leading and Following: s-s

Worst
case-dimensional

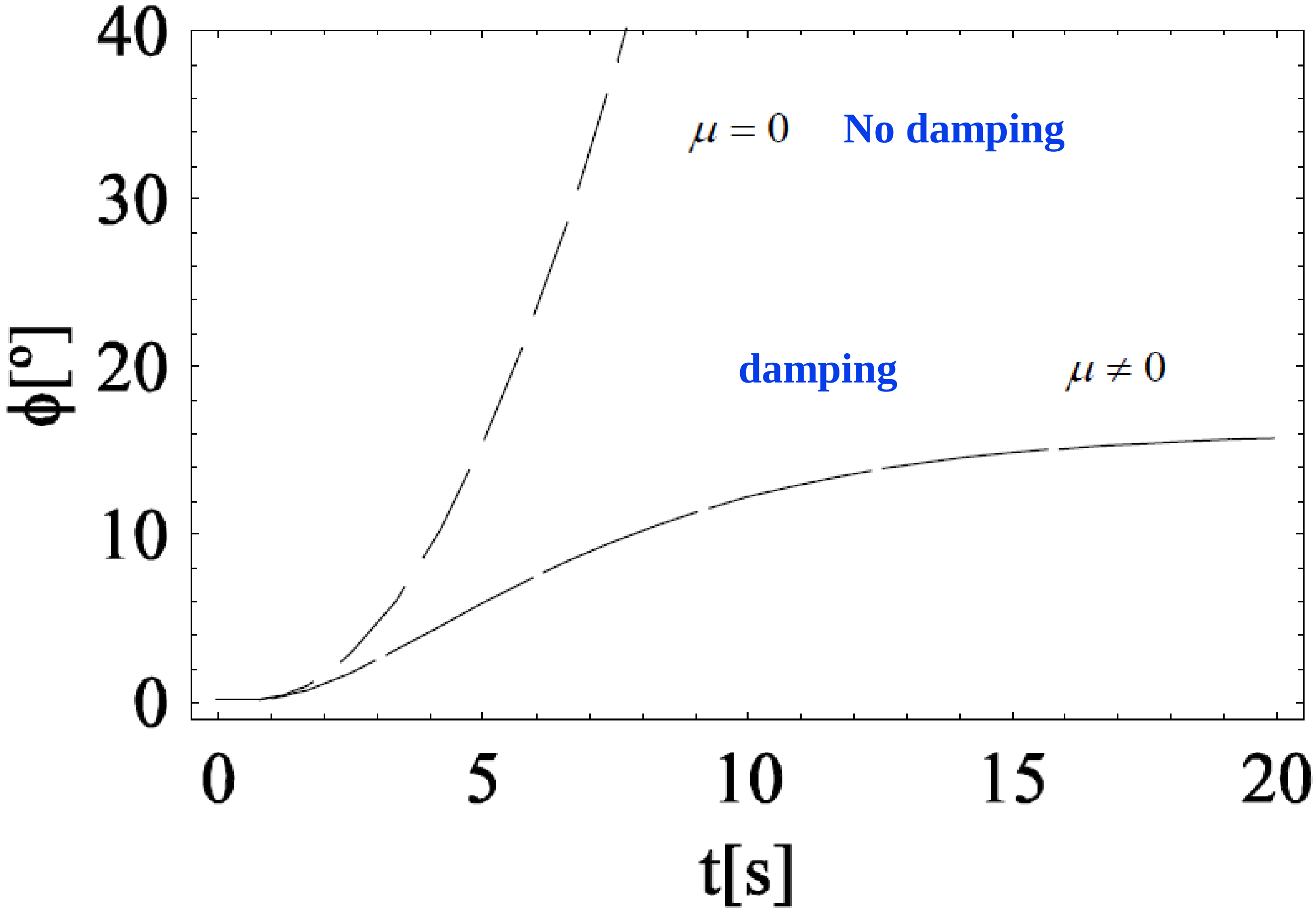


Peak roll rate:
2.35º/s



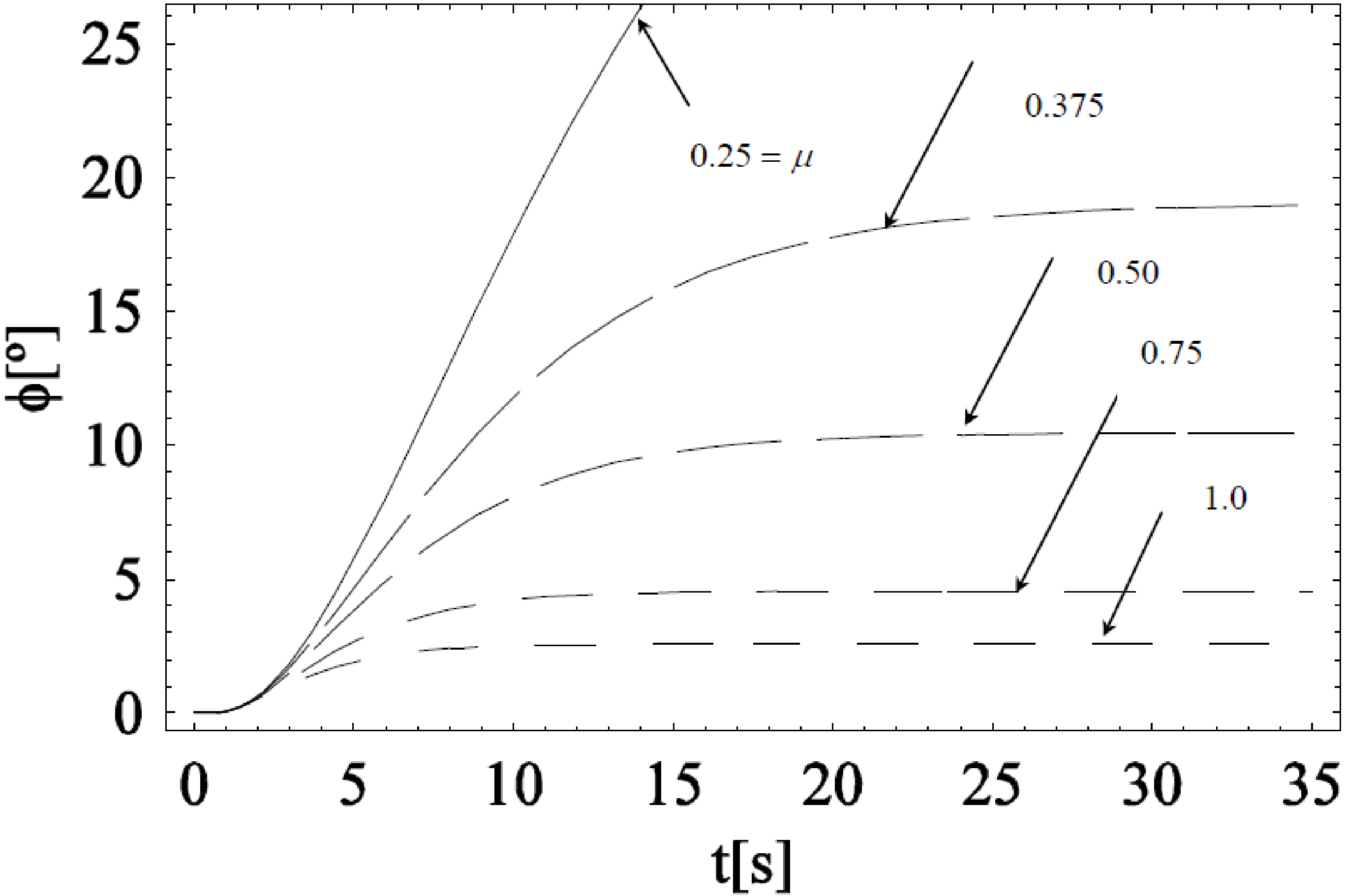
Asymptotic
bank angle
12.4º

Without and with Roll Damping



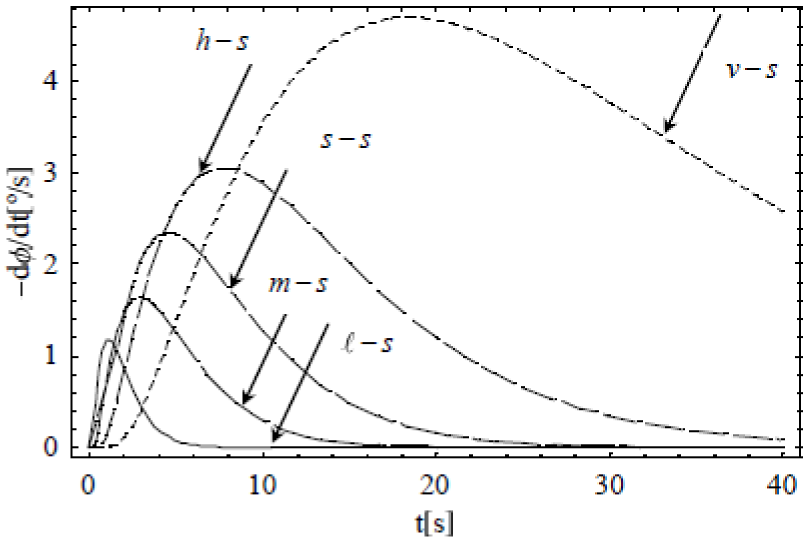
Weak and Strong Roll Damping

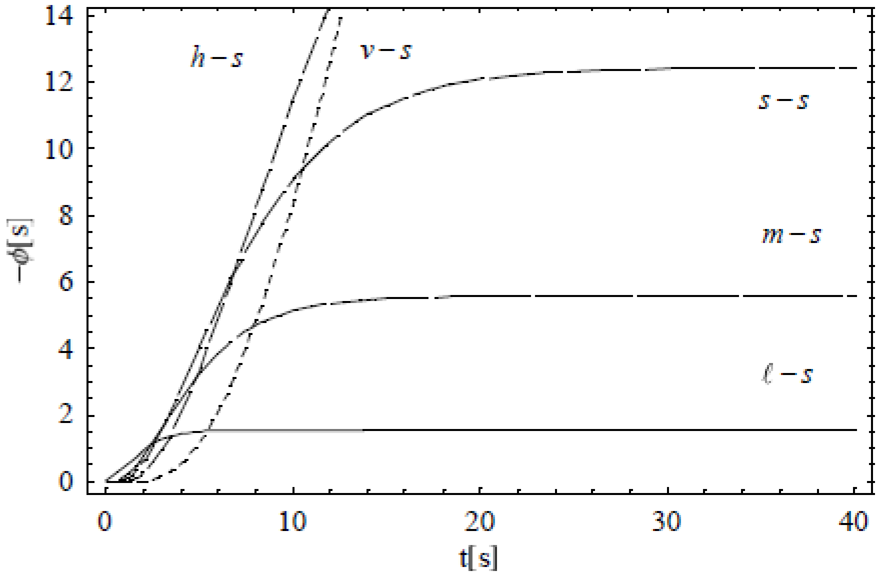
Weak and strong roll damping



MORE DAMPING SMALLER BANK ANGLE

Special (s) following aircraft





Leading Aircraft

l – light: Jetstar

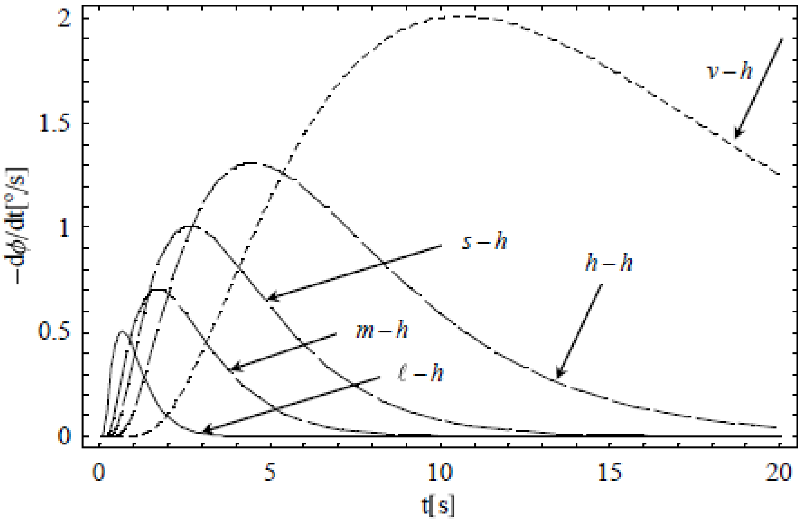
m – medium: B737 – 200

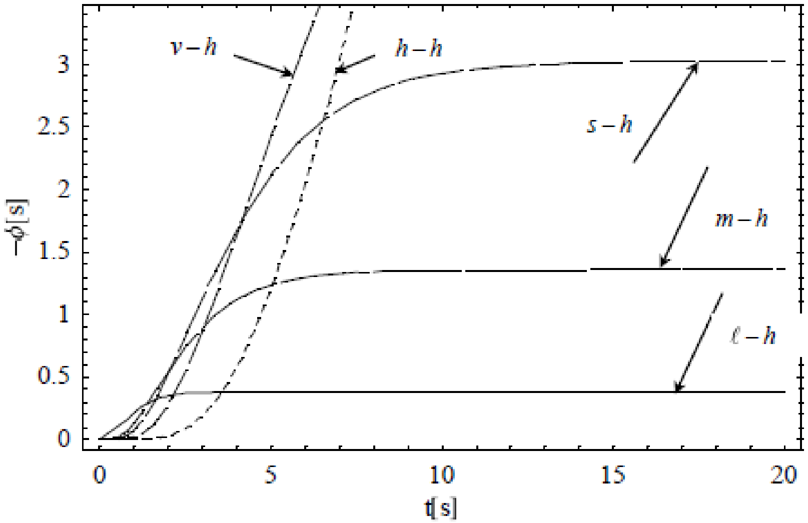
s – special; B757 – 200

h – heavy: B747 – 100

v – very large: A380 – 100

Heavy (h) following aircraft





Leading Aircraft

l – light: Jetstar

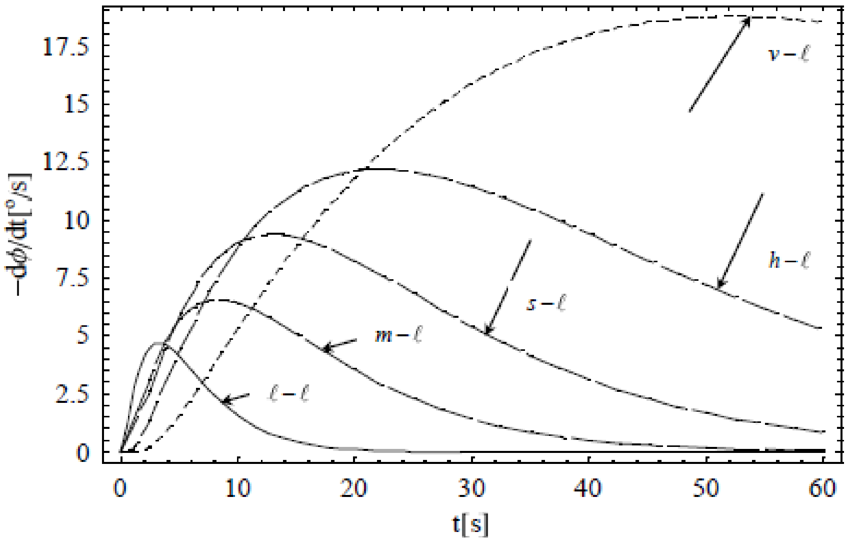
m – medium: B737 – 200

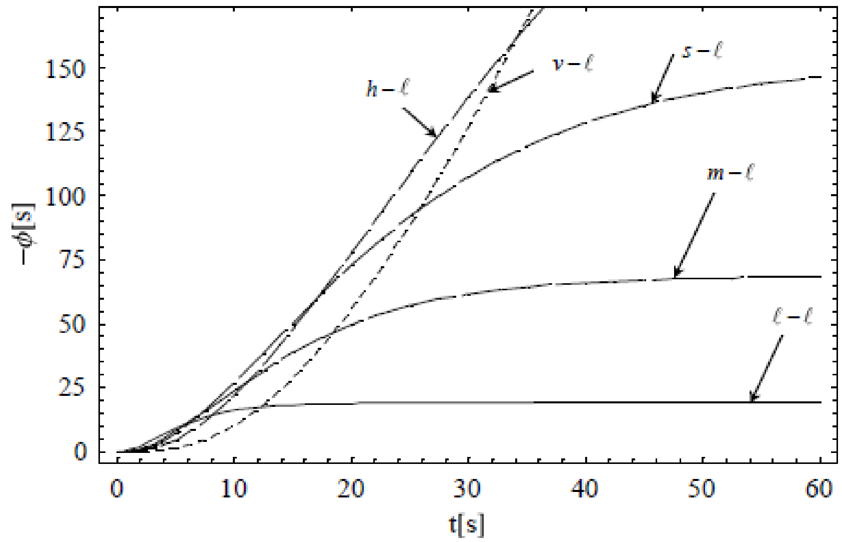
s – special; B757 – 200

h – heavy: B747 – 100

v – very large: A380 – 100

Light (l) following aircraft





Leading Aircraft

l – light: Jetstar

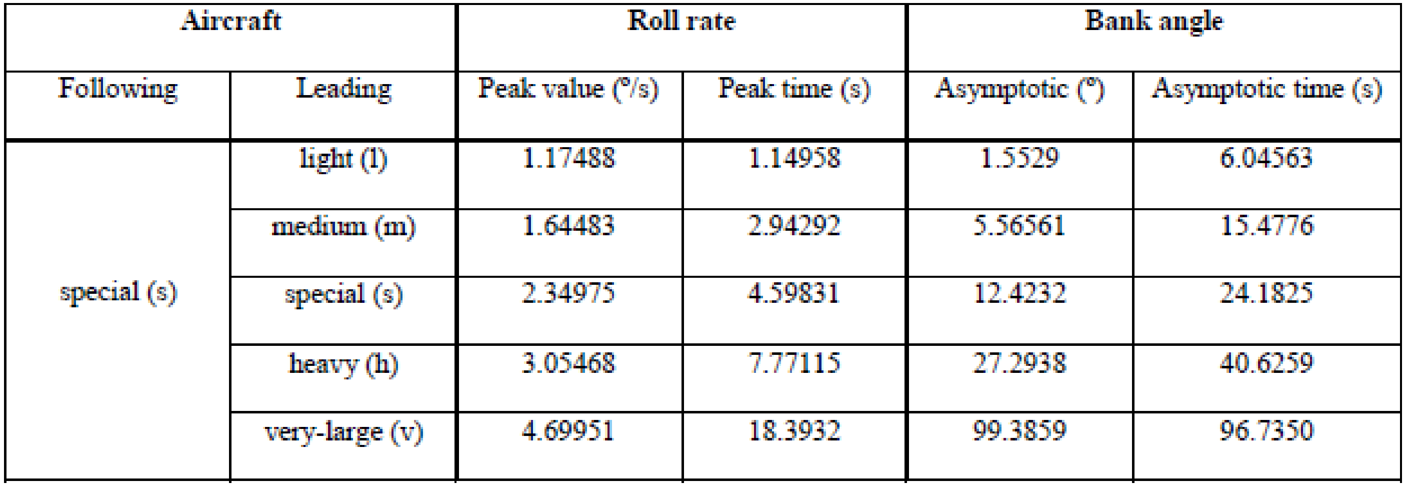
m – medium: B737 – 200

s – special; B757 – 200

h – heavy: B747 – 100

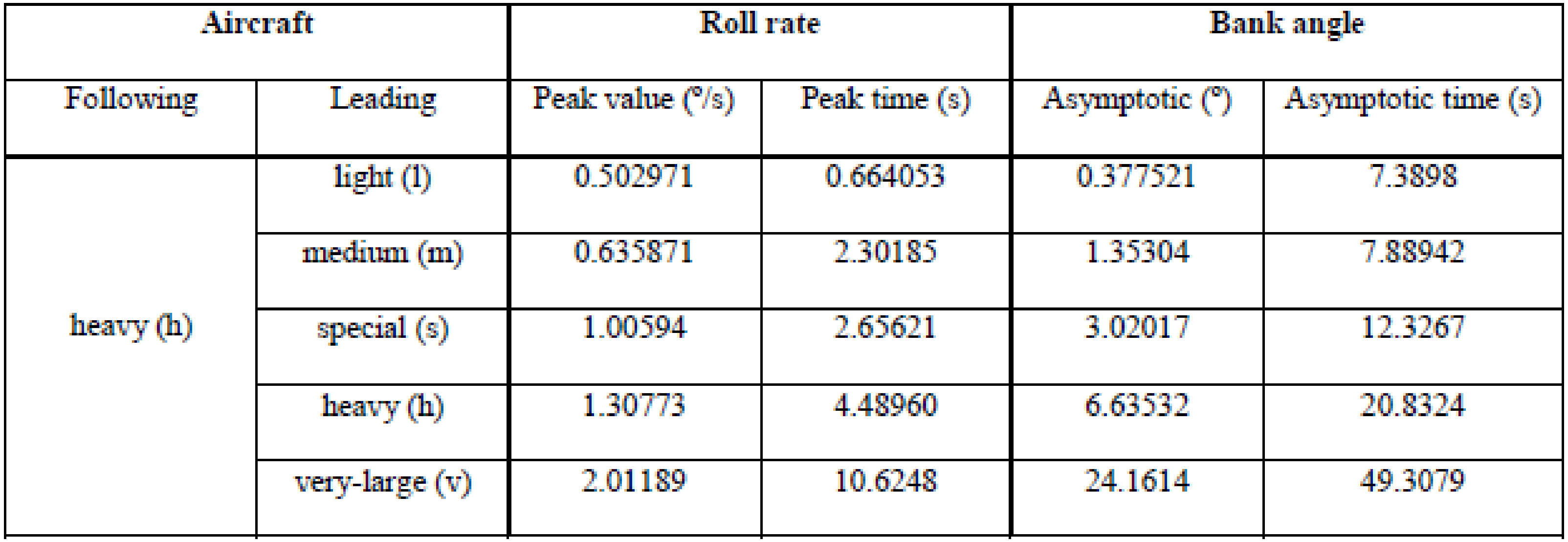
v – very large: A380 – 100

Table I – Wake vortex Effects on Following Special Aircraft



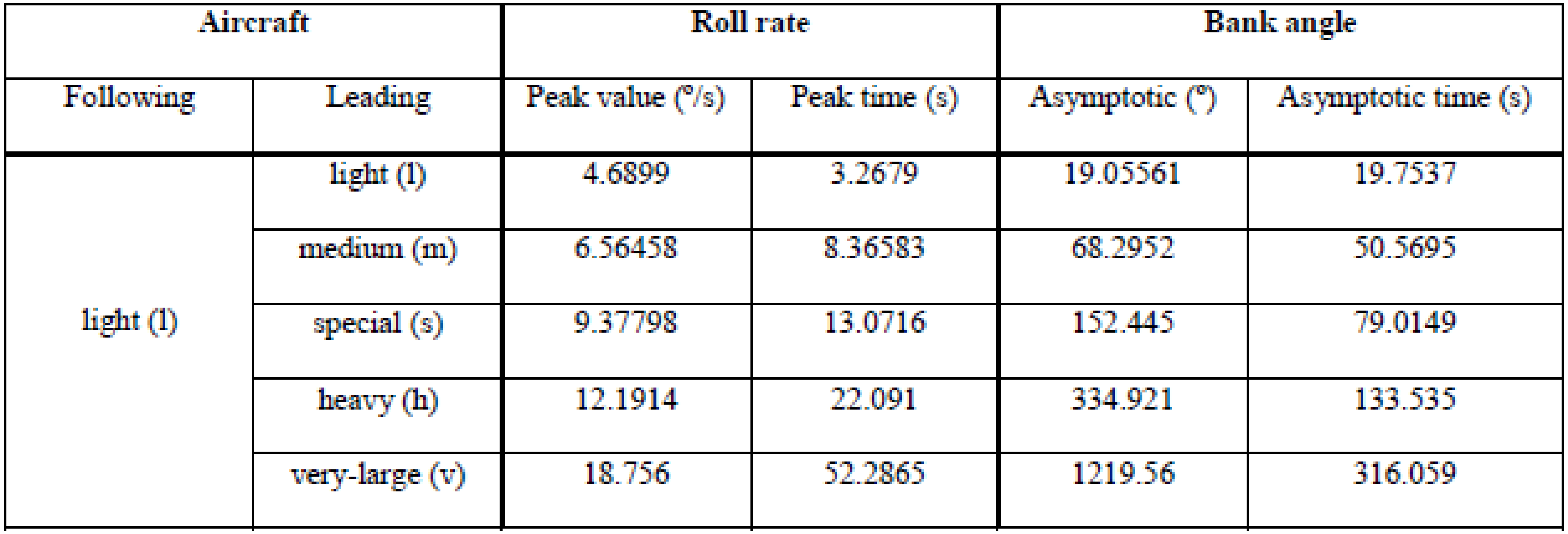
Aircraft		Roll rate		Bank angle	
Following	Leading	Peak value (°/s)	Peak time (s)	Asymptotic (°)	Asymptotic time (s)
special (s)	light (l)	1.17488	1.14958	1.5529	6.04563
	medium (m)	1.64483	2.94292	5.56561	15.4776
	special (s)	2.34975	4.59831	12.4232	24.1825
	heavy (h)	3.05468	7.77115	27.2938	40.6259
	very-large (v)	4.69951	18.3932	99.3859	96.7350

Table II – Wake vortex Effects on Following Heavy Aircraft



Aircraft		Roll rate		Bank angle	
Following	Leading	Peak value (°/s)	Peak time (s)	Asymptotic (°)	Asymptotic time (s)
heavy (h)	light (l)	0.502971	0.664053	0.377521	7.3898
	medium (m)	0.635871	2.30185	1.35304	7.88942
	special (s)	1.00594	2.65621	3.02017	12.3267
	heavy (h)	1.30773	4.48960	6.63532	20.8324
	very-large (v)	2.01189	10.6248	24.1614	49.3079

Table III – Wake vortex Effects on Following Light Aircraft



Aircraft		Roll rate		Bank angle	
Following	Leading	Peak value (°/s)	Peak time (s)	Asymptotic (°)	Asymptotic time (s)
light (l)	light (l)	4.6899	3.2679	19.05561	19.7537
	medium (m)	6.56458	8.36583	68.2952	50.5695
	special (s)	9.37798	13.0716	152.445	79.0149
	heavy (h)	12.1914	22.091	334.921	133.535
	very-large (v)	18.756	52.2865	1219.56	316.059

CONCLUSIONS (1/2)

- The roll response due to the wake vortex effects can be calculated for any pair of aircraft;
- The response is the same for all pairs of aircraft using suitable dimensionless variables;
- Using the parameters for each aircraft specifies the dimensionless roll rate and bank angle as a function of time;
- Peak roll rate and the time of its occurrence can be calculated;

CONCLUSIONS (2/2)

- Maximum bank angle can be determined and depends on damping to stop the rolling motion;
- Initial conditions (initial bank angle and roll rate) can be combined with aileron deflection and wake vortex effects;
- Wake vortex effects can be calculated for any separation between leading and following aircraft;
- Parametric dependences on aircraft characteristics and flight conditions are identified;
- Atmospheric model is simplified and is main limitation.